

Solving Quadratic Equations with Square Roots

Solve each equation by taking square roots.

1) $k^2 = 76$

2) $k^2 = 16$

3) $x^2 = 21$

4) $a^2 = 4$

5) $x^2 + 8 = 28$

6) $2n^2 = -144$

7) $-6m^2 = -414$

8) $7x^2 = -21$

9) $m^2 + 7 = 88$

10) $-5x^2 = -500$

11) $-7n^2 = -448$

12) $-2k^2 = -162$

13) $x^2 - 5 = 73$

14) $16n^2 = 49$

$$15) n^2 - 5 = -4$$

$$16) n^2 + 8 = 80$$

$$17) 7v^2 + 1 = 29$$

$$18) 10n^2 + 2 = 292$$

$$19) 2m^2 + 10 = 210$$

$$20) 9n^2 + 10 = 91$$

$$21) 5n^2 - 7 = 488$$

$$22) 8n^2 - 6 = 306$$

$$23) 10n^2 - 10 = 470$$

$$24) 8n^2 - 4 = 532$$

$$25) 4r^2 + 1 = 325$$

$$26) 8b^2 - 7 = 193$$

$$27) 2k^2 - 2 = 144$$

$$28) 3 - 4x^2 = -85$$

Solving Quadratic Equations with Square Roots

Solve each equation by taking square roots.

1) $k^2 = 76$

$\{8.717, -8.717\}$

2) $k^2 = 16$

$\{4, -4\}$

3) $x^2 = 21$

$\{4.582, -4.582\}$

4) $a^2 = 4$

$\{2, -2\}$

5) $x^2 + 8 = 28$

$\{4.472, -4.472\}$

6) $2n^2 = -144$

No solution.

7) $-6m^2 = -414$

$\{8.306, -8.306\}$

8) $7x^2 = -21$

No solution.

9) $m^2 + 7 = 88$

$\{9, -9\}$

10) $-5x^2 = -500$

$\{10, -10\}$

11) $-7n^2 = -448$

$\{8, -8\}$

12) $-2k^2 = -162$

$\{9, -9\}$

13) $x^2 - 5 = 73$

$\{8.831, -8.831\}$

14) $16n^2 = 49$

$\{1.75, -1.75\}$

15) $n^2 - 5 = -4$

$\{1, -1\}$

16) $n^2 + 8 = 80$

$\{8.485, -8.485\}$

17) $7v^2 + 1 = 29$

$\{2, -2\}$

18) $10n^2 + 2 = 292$

$\{5.385, -5.385\}$

19) $2m^2 + 10 = 210$

$\{10, -10\}$

20) $9n^2 + 10 = 91$

$\{3, -3\}$

21) $5n^2 - 7 = 488$

$\{9.949, -9.949\}$

22) $8n^2 - 6 = 306$

$\{6.244, -6.244\}$

23) $10n^2 - 10 = 470$

$\{6.928, -6.928\}$

24) $8n^2 - 4 = 532$

$\{8.185, -8.185\}$

25) $4r^2 + 1 = 325$

$\{9, -9\}$

26) $8b^2 - 7 = 193$

$\{5, -5\}$

27) $2k^2 - 2 = 144$

$\{8.544, -8.544\}$

28) $3 - 4x^2 = -85$

$\{4.69, -4.69\}$

Notes for Solving Quadratic Equations using Square Roots

Solving $x^2 = 36$ and similar equations

Suppose we want to solve the equation $x^2 = 36$. Let's first verbalize what the equation is asking us to find. It is asking us *which number, when multiplied by itself, equals 36*.

If this question sounds familiar to you, it's because this is the definition of the square root of 36, which is expressed mathematically as $\sqrt{36}$.

Now, this is how the complete solution of the equation looks:

$$x^2 = 36$$

$$\sqrt{x^2} = \sqrt{36} \quad \text{Take the square root.}$$

$$x = \pm\sqrt{36}$$

$$x = \pm 6$$

What the \pm sign means

Note that every positive number has *two* square roots: a positive square root and a negative square root. For example, both 6 and -6 , when squared, equal 36. Therefore, this equation has *two solutions*.

The \pm is just an efficient way of representing this concept mathematically. For example, ± 6 means "either 6 or -6 ".

Solving $(x - 2)^2 = 49$ and similar equations

Here is how the solution of the equation $(x - 2)^2 = 49$ goes:

$$(x - 2)^2 = 49$$

$$\sqrt{(x - 2)^2} = \sqrt{49} \quad \text{Take the square root.}$$

$$x - 2 = \pm 7$$

$$x = \pm 7 + 2 \quad \text{Add 2.}$$

Therefore, the solutions are $x = 9$ and $x = -5$.

Another Example:

$$2x^2 + 3 = 131$$

$$2x^2 = 128 \quad \text{Subtract 3.}$$

$$x^2 = 64 \quad \text{Divide by 2.}$$

$$\sqrt{x^2} = \sqrt{64} \quad \text{Take the square root.}$$

$$x = \pm 8$$

Quadratic Formula Notes:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1: Solve $12x^2 + 7x = 12$

Step 1: Simplify the problem to get the problem in the form $ax^2 + bx + c = 0$.

$$12x^2 + 7x = 12 \rightarrow 12x^2 + 7x - 12 = 0$$

Step 2: Identify the values of a, b, and c, then plug them into the quadratic formula.

$$a = 12, b = 7, \text{ and } c = -12$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow x = \frac{-7 \pm \sqrt{7^2 - 4(12)(-12)}}{2(12)}$$

Step 3: Simplify the numbers within the quadratic formula.

$$x = \frac{-7 \pm \sqrt{7^2 - 4(12)(-12)}}{2(12)} \rightarrow x = \frac{-7 \pm \sqrt{625}}{24}$$

Step 4: Simplify the radical and reduce/simplify to get the final answer.

$$x = \frac{-7 \pm \sqrt{625}}{24} \rightarrow x = \frac{-7 \pm 25}{24} \rightarrow x = \frac{3}{24} \text{ or } x = \frac{4}{-24}$$

Note: In this example the radical disappeared and the final answers were simple fractions which means that this problem could have been solved by factoring, but since we already have the answer at this point it makes little difference.

Example 2 – Solve: $-3x^2 + 6x + 5 = -5$

<p>Step 1: To use the quadratic formula, the equation must be equal to zero, so move the -5 back to the left hand side.</p>	$-3x^2 + 6x + 5 = 0$
<p>Step 2: Identify a, b, and c and plug them into the quadratic formula. In this case $a = -3$, $b = 6$, and $c = 5$.</p>	$x = \frac{-6 \pm \sqrt{6^2 - 4(-3)(5)}}{2(-3)}$
<p>Step 3: Use the order of operations to simplify the quadratic formula.</p>	$x = \frac{-6 \pm \sqrt{36 + 60}}{-6} = \frac{-6 \pm \sqrt{96}}{-6}$
<p>Step 4: Simplify the radical, if you can. In this case you can simplify the radical into:</p>	$x = \frac{-6 \pm 4\sqrt{6}}{-6}$
<p>Step 5: Reduce the problem, if you can. In this case you can reduce the entire problem by 2.</p>	$x = \frac{-3 \pm 2\sqrt{6}}{-3} \quad \text{or} \quad x = \frac{3 \pm 2\sqrt{6}}{3}$

Example 5 – Solve: $6x^2 - 13x = 8$

<p>Step 1: To use the quadratic formula, the equation must be equal to zero, so move the 8 back to the left hand side.</p>	$6x^2 - 13x - 8 = 0$
<p>Step 2: Identify a, b, and c and plug them into the quadratic formula. In this case $a = 6$, $b = -13$, and $c = -8$.</p>	$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(6)(-8)}}{2(6)}$
<p>Step 3: Use the order of operations to simplify the quadratic formula.</p>	$x = \frac{13 \pm \sqrt{169 + 192}}{12} = \frac{13 \pm \sqrt{361}}{12}$
<p>Step 4: Simplify the radical, if you can. In this case you can simplify the radical into:</p>	$x = \frac{13 \pm 19}{12}$
<p>Step 5: Since this problem does not contain any square roots, you can simplify the final answer into:</p>	$x = \frac{13 + 19}{12} \quad \text{or} \quad x = \frac{13 - 19}{12}$ $x = \frac{32}{12} \quad \text{or} \quad x = \frac{-6}{12}$
<p>Step 6: Reduce each answer individually and you should get:</p>	$x = \frac{8}{3} \quad \text{or} \quad x = -\frac{1}{2}$

Practice Problems:

1) $m^2 - 5m - 14 = 0$

2) $b^2 - 4b + 4 = 0$

3) $2m^2 + 2m - 12 = 0$

4) $2x^2 - 3x - 5 = 0$

5) $x^2 + 4x + 3 = 0$

6) $2x^2 + 3x - 20 = 0$

$$7) 4b^2 + 8b + 7 = 4$$

$$8) 2m^2 - 7m - 13 = -10$$

$$9) 2x^2 - 3x - 15 = 5$$

$$10) x^2 + 2x - 1 = 2$$

$$11) 2k^2 + 9k = -7$$

$$12) 5r^2 = 80$$

Answers:

$$1) m^2 - 5m - 14 = 0$$

$$\{7, -2\}$$

$$2) b^2 - 4b + 4 = 0$$

$$\{2\}$$

$$3) 2m^2 + 2m - 12 = 0$$

$$\{2, -3\}$$

$$4) 2x^2 - 3x - 5 = 0$$

$$\left\{\frac{5}{2}, -1\right\}$$

$$5) x^2 + 4x + 3 = 0$$

$$\{-1, -3\}$$

$$6) 2x^2 + 3x - 20 = 0$$

$$\left\{\frac{5}{2}, -4\right\}$$

$$7) 4b^2 + 8b + 7 = 4$$

$$\left\{-\frac{1}{2}, -\frac{3}{2}\right\}$$

$$8) 2m^2 - 7m - 13 = -10$$

$$\left\{\frac{7 + \sqrt{73}}{4}, \frac{7 - \sqrt{73}}{4}\right\}$$

$$9) 2x^2 - 3x - 15 = 5$$

$$\{4, -\frac{5}{2}\}$$

$$10) x^2 + 2x - 1 = 2$$

$$\{1, -3\}$$

$$11) 2k^2 + 9k = -7$$

$$\left\{-1, -\frac{7}{2}\right\}$$

$$12) 5r^2 = 80$$

$$\{4, -4\}$$